Pumping in an Infinite Confined Aquifer - The Theis Solution

Theis (1935) presented an exact analytical solution for the transient drawdown in an infinite uniform confined aquifer (See Fig 1).

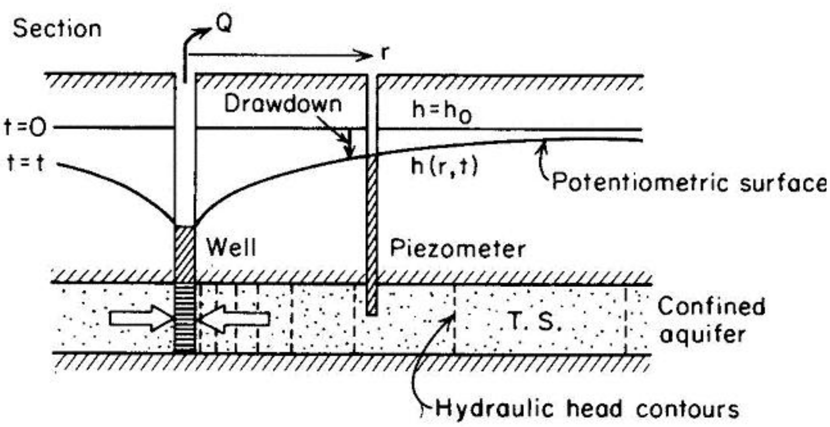

Fig 1. Radial flow to a well in a horizontal confined aquifer (Freeze and Cherry, 1979)

Analytical Solution

The analytical solution of the drawdown as a function of time and distance is expressed by equation (1):

$$h_0 - h(x, y, t) = \frac{Q}{4\pi T} W(u) \qquad (1)$$

Where
$$u = \frac{(x^2 + y^2)S}{4Tt} \qquad (2)$$

and

h_0	is the constant initial hydraulic head, [L]
Q	is the constant flow rate abstracted from the well , [L^3/T]
S	is the aquifer storage coefficient [-]
x, y	is the distance at any time after the start of pumping,[L]
T	is the aquifer Transmissivity, [L^2/T]
t	is the time,[T]

IGW Numerical Solution

IGW is applied to solve a flow problem for the following situation:

Physical Parameters

Q = 1000 m^3/day
h_0 = 20 m

$S = 0.0002$
$t = 0.00175$ days $=151.37$ sec
$T = 1000\ \mathrm{m^2/day} = K_x \cdot \text{Thickness} = 50\ \mathrm{m/day} \cdot 20\ \mathrm{m}$

Numerical Parameters:

$\Delta x = 10\mathrm{m}$
$\Delta y = 10\mathrm{m}$
$\Delta t = 1.036$ sec

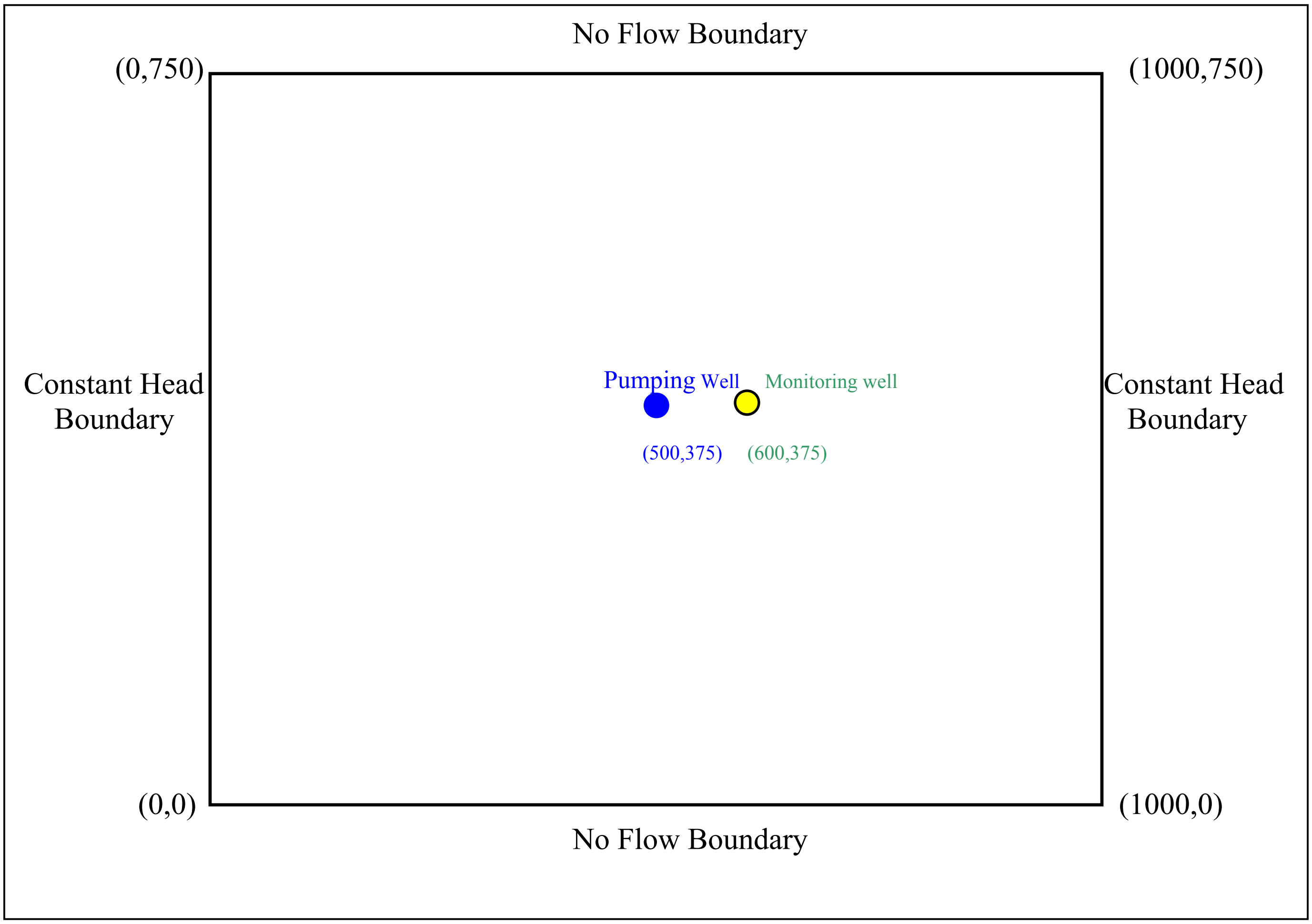

Fig 2. Plan view of IGW model set up for comparison to the Theis solution

Analytical Solution versus IGW

The IGW solutions are presented and compared with the exact solution in Figures 3 and 4.

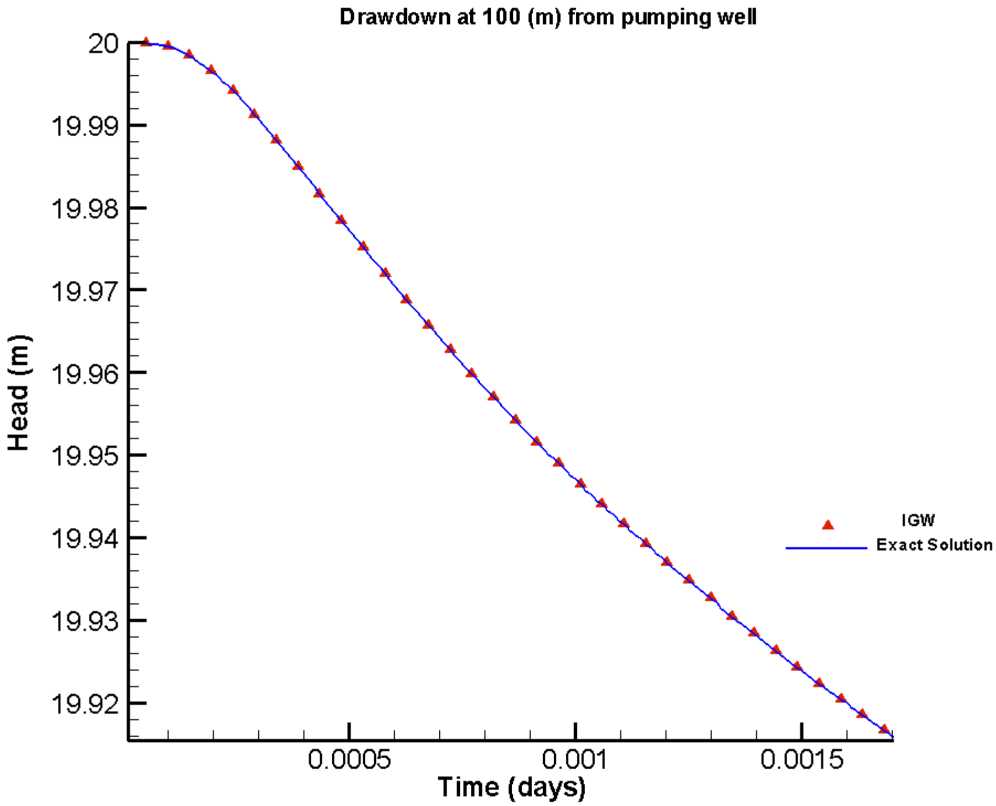

Fig 3. Comparison of the exact and predicted hydrograph using IGW at a location 100 meters from the well.

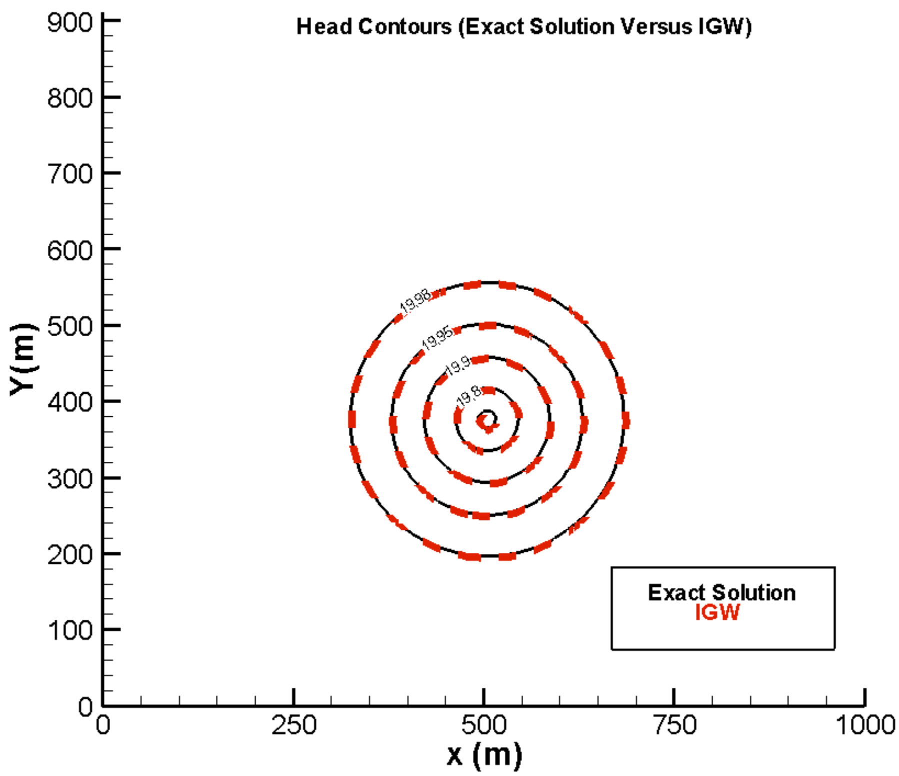

Fig 4. Comparison of the Theis solution and with IGW solution at151.37 seconds

The numerical solution is graphically indistinguishable from the exact solution until the drawdown influence begins to reach the boundaries.

The cone of depression is also plotted in 3D in Figure 5.

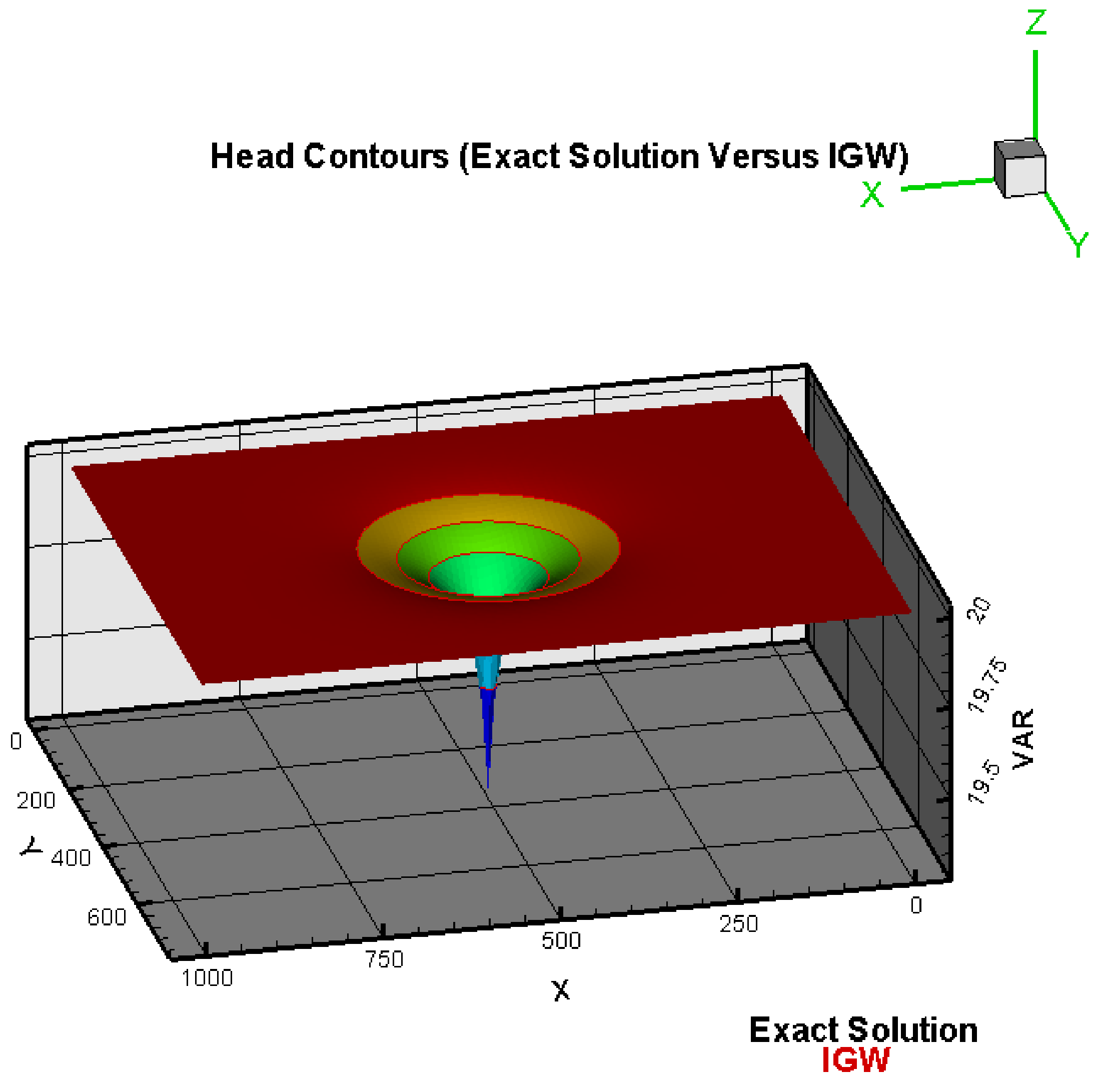

Figure 5. Cone of Depression